

BULETINUL INSTITUTULUI POLITEHNIC DIN IAȘI  
Publicat de  
Universitatea Tehnică „Gheorghe Asachi” din Iași  
Tomul LIX (LXIII), Fasc. 5, 2013  
Secția  
CONSTRUCȚII. ARHITECTURĂ

## APPLICATION OF NUMERICAL METHODS IN DESIGN OF EMBEDDED RETAINING STRUCTURES ACCORDING TO EUROCODE 7

BY

IANCU-BOGDAN TEODORU\*, ANGHEL STANCIU, IRINA LUNGU  
and FLORIN BEJAN

“Gheorghe Asachi” Technical University of Iași  
Faculty of Civil Engineering and Building Services

Received: December 3, 2013

Accepted for publication: December 14, 2013

**Abstract.** Embedded retaining structures are commonly designed based on the limit equilibrium concept. This approach may sometimes leads to significant shortcomings, mainly because it is difficult, often impossible, to estimate the effect that soil movement may have on adjacent buildings or services. Contrary to the traditional design methods, and as an alternative to them, numerical solutions, now based mostly upon finite element and finite difference techniques, can provide the engineer with all the desired design information in a consistent manner, due to its comprehensive modeling capability (*e.g.* complex soil behavior, practically any type of geometry and loading conditions). Application of numerical methods for ultimate limit state design in accordance with Eurocode 7 is highlighted in the present paper. Some conclusive remarks are made regarding the numerical modeling in a finite element environment (choosing and defining the model geometry, calibration, etc).

**Key words:** excavation; Eurocode 7; retaining structures; numerical analysis.

### 1. Introduction

Construction works, especially those in the urban areas, often require, due to the limited space, the execution of the related excavations near the

---

\* Corresponding author: *e-mail*: bteodoru@ce.tuiasi.ro

existing facilities. Therefore, in order to avoid unfavourable ground deformations that can affect adjacent structures, the excavations are done under the protection of the earth-retaining structures, the current type being the embedded retaining walls.

Calculation procedures adopted in current design practice of the earth-retaining structures are based on simplified methods supported either by the limit equilibrium concept (Coulomb theory), or by the stress field solution (Rankine theory); the limit analysis and the beam-spring approach (Desai & Christian, 1977; Potts & Zdravkovic, 1999) are also simplified models, but in a general context these have a limited use. This approach is limitative and lacking of information needed for an accurate design. For example, the data currently related to the excavation-induced ground movement are very limited and mainly empirically supported; a direct consequence of this shortcoming is that the evaluation of the potential risk (especially in the urban areas) upon the neighboring buildings, could not be quantified.

Another reason why this approach may be inadequate, is that the flexibility of the structure does not enter the analysis but it will certainly influence the way in which the retaining structure deforms and thus, the earth pressures mobilization.

Contrary to the traditional practices and as an alternative to them, the numerical solutions based on the Finite Element Method (FEM) could provide in a consistent manner, due to its modeling capability, any data needed to achieve an accurate design.

## **2. Numerical Modeling Using Finite Element Technique**

### **2.1. Model Geometry**

FEM does not inherently includes precise rules on finite elements mesh design of a given structure or domain. For this reason, to approximate the best solution of a certain model, the discretization must be based on a careful analysis of the state of stress and strain associated with the real model. Thus some particularities, especially those relating to the geometric or material discontinuities, boundary and loading conditions will be accounted for. In other words, the mesh design is strictly related to the problem under investigation and, therefore, there is no predefined rule for its generation, depending rather on the user expertise and creativity.

Hence, the terms of *fine* or *rough* discretisation becomes relative; the discretisation is sufficiently fine if the one performing the analysis has experience or has already performed modeling with finer discretisation and the differences between the obtained results are insignificant (Desai & Christian, 1977; Potts & Zdravkovic, 1999; Popa, 2004). A compromise between the available computation capacity and the required accuracy of the results is to divide the analysed domain into sub-domains with different mesh sizes for each

one. The sub-domains will be defined near any geometrical or material discontinuities (*e.g.* soil layers) or near any applied load where the stress and displacement gradients are expected to be large.

Generally speaking, the numerical model is reduced to a domain where the boundaries have negligible displacements. Defining these boundaries is a delicate task and the associated rules have no general nature, remaining to be supported by the obtained solution. Thus, the results must verify that the perturbations caused by the adopted boundary conditions are insignificant (*e.g.* a restrained displacement must not generate tension, compression or shearing stresses – Popa, 2004).

## 2.2. Types of Analyses

In general, any geotechnical engineering problem and, in particular, those involving retaining structures, are three-dimensional ones and, therefore, only the three-dimensional analyses shall entirely represent the real nature of the studied case. For practical reasons, coupling the analysed problem with the plane strain conditions is another compromise between the available computation capacity and the required results accuracy. It is clear that the model dimensions and the number of finite elements needed are thus reduced. However, in practice, the retaining structures do not always fulfil the plane strain conditions, *e.g.*, braced or anchored walls. Commonly, with errors below 15% against the three-dimensional case, these systems are also analysed assuming the plane strain condition (Desai & Christian, 1977). The boundary conditions (*e.g.* construction-sequence, anchorage/strut execution, etc.) are also not applied strictly two-dimensionally but, rather, three-dimensionally. These are just few aspects due to whom, for a studied case that furthers from the assumption of plane strain conditions, the displacements are overestimated compared to the ones resulted from the three-dimensional analyses (Desai & Christian, 1977).

## 2.3. Constitutive Models

Constitutive models must be set-up for both the soil and the structure. Generally, in soil-structure applications the material may be assumed to be linear elastic, nonlinear elastic or with various elasto-plastic behaviors. However, adopting a constitutive model for the soil behavior for its loading and unloading response must consider the predominant nature of the problem under investigation. For example, if the aim is to obtain a solution in terms of behavior of the soil brought in a failure condition, then the adopted model shall have to simulate, first of all, as realistically as possible (also) the soil strength mobilization; if the ground movements, especially those adjacent to the retaining structure, are analysed, then it is recommended to use a model that captures the non-linear soil behavior under small deformations (Potts & Zdravkovic, 1999).

A linear elastic behavior is usually applied for the structure.

#### 2.4. Initial Stress Condition

This step is mandatory in any numerical modeling of the soil behavior under specific loading conditions, having in view the direct dependency of the reactive forces on the stress conditions controlled by gravity or arising from the various geological processes. The generation of the initial stress conditions could be achieved either by modeling the geological processes or by setting values the effective stress and the pore water pressure (Potts & Zdravkovic, 1999; Popa, 2004).

#### 2.5. Structure Modeling

Embedded walls could be modeled by using either solid-type finite elements, beam-type elements or plate-type. And yet, using the last type of element, for it has no thickness, involves a certain degree of simplification that allows overrating the lateral displacements and the bending moments (Potts & Zdravkovic, 2001). On the other hand, by using solid-type finite elements, it becomes more complicated (but not impossible) to achieve the values of the internal forces: bending moments, axial and shear forces; thus additional calculations are needed, as the values of engineering interest are obtained following numerical integration techniques.

### 3. Example: Excavation Supported by Cantilevered Wall

This simple excavation example is intended to illustrate the application of finite element analysis to the design of embedded retaining structures. It also provides general guidance on using Eurocode 7 Design Approach with the finite element model.

The finite element modeling comprises two-dimensional plane strain analyses carried out by using PLAXIS software package. PLAXIS is a specially developed finite element program to perform geotechnical analysis of deformation and stability of soil structures, as well as groundwater and heat flow, in geo-engineering applications such as excavations, foundations, embankments and tunnels (Brinkgreve *et al.*, 2006).

#### 3.1. Geometry

Fig. 1 shows the simple example of an embedded retaining wall for an excavation having the nominal depth  $H_{\text{nom.}} = 5.0$  m into a cohesionless homogeneous soil layer; an additional depth,  $\Delta H = 0.1H_{\text{nom.}} = 0.5$  m, is considered in order to allow for accidental overdig, giving a design value of the excavation depth  $H = 5.50$  m. A characteristic variable surcharge,  $q_k = 15$  kPa, acts on the ground surface behind the wall.

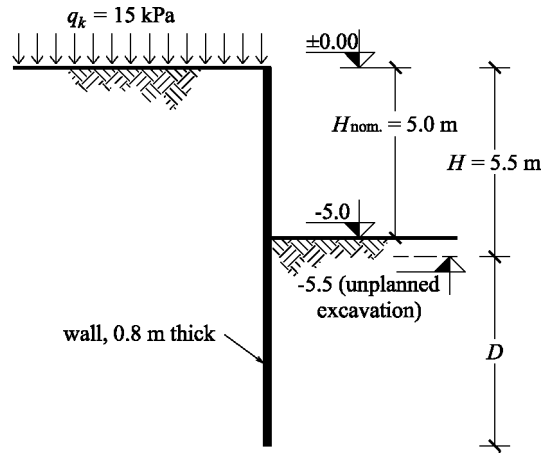


Fig. 1. – Basic geometry of the embedded wall.

### 3.2. Mesh and Boundary Conditions

The model geometry is introduced by means of the finite element mesh shown in Fig. 2. It consists of approximately 600 15-noded triangular elements with a fourth order interpolation for displacements and twelve Gauss points for the numerical integration. The parts of the domain where high stress gradients can be expected have been modeled with smaller elements. The retaining structure was modeled by using five-node beam elements. To allow relative displacements between the wall structure and the soil body, interface elements were used along the soil–wall contact.

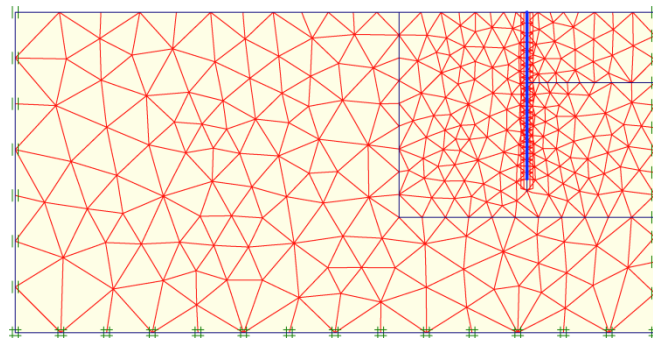


Fig. 2. – The finite element mesh.

The finite element mesh dimensions are 25 m deep and 50 m wide. To restrain the domain against relative rigid body motion, the bottom boundary of the mesh is fixed in both horizontal and vertical directions and the vertical boundaries are fixed in horizontal direction.

### 3.3. Material Parameters

The soil is assumed to behave as an elastic perfectly plastic material, defined by linear material properties and the Mohr-Coulomb constitutive law. Its characteristic properties, in terms of model parameters, are given in Table 1.

**Table 1**  
*Soil Properties*

Parameter	Value	Unit
Saturated unit weight, $\gamma_{\text{sat}}$	20	kN/m <sup>3</sup>
Unsaturated unit weight, $\gamma_{\text{unsat}}$	20	kN/m <sup>3</sup>
Friction angle, $\phi'$	30	°
Cohesion, $c'$	0.1	kPa
Dilatancy angle, $\psi$	0	°
Young's modulus, $E_{\text{ref}}$	30	MPa
Poisson's ratio, $\nu$	0.20	-
Interface strength ratio, $R_{\text{inter}}$	0.67	-

The wall is assumed to behave as isotropic linear elastic material having Young's modulus  $E = 30,000 \text{ MN/m}^2$ , Poisson's ratio  $\nu = 0.2$  and unit weight  $\gamma = 25 \text{ kN/m}^3$ .

### 3.4. Initial Stress Conditions

*Prior* to the main calculations, the initial condition of the soil must be determined. This includes both the initial stress state and the initial water pressures within the soil. The initial stress state was prescribed in terms of the earth pressure coefficient at rest,  $k_0 = 1 - \sin\phi'$ , and fully drained conditions were assumed, with zero pore pressures everywhere.

### 3.5. Design Approaches with Eurocode 7

Eurocode 7 allows for three different Design Approaches (DA) for Ultimate Limit State (ULS) designs in persistent and transient situations (Table 2); particularly Section 9 treats retaining structures (EN 1997-1, 2004). As states Schweiger, (2005), they differ in the way the partial factors of safety are applied to soil properties, resistance and actions (Tables 3 and 4).

Design Approach 1 (DA1) may be termed an action and material factor approach using two combinations (C1 to C2) of sets of partial factors mainly applied at the source, as load and material factors. When load factors applied at the source lead to physically impossible situations, they may be applied to the effects of the actions. Design values of pile and anchor resistances are obtained by applying the partial factors on their (measured or calculated) resistance. Combination 1 (DA1-C1) usually governs the structural design and it focuses on the design against unfavourable variability of actions, while design values of

ground properties are equal to their characteristic values. Combination 2 (DA1-C2) governs the geotechnical design, factoring less restrictively actions but factoring ground strength parameters and resistances (Frank *et al.*, 2004).

Design Approach 2 (DA2) may be termed an action (effect) and resistance factor approach and it requires a single calculation where partial factors are applied to the ground resistance and the actions or to the ground resistance and the effects of the actions. According to Frank *et al.*, (2004), the partial safety factors are applied to the characteristic actions at the beginning of the calculation for DA2, whereas they are applied to the characteristic effect of actions at the end of the calculations for DA2\*.

Design Approach 3 (DA3) may be termed an action (effect) and material factor approach and it requires a single calculation where partial factors are applied to actions (effect) from the structure and to ground strength parameters simultaneously, while geotechnical actions and resistance are left mainly unfactored (Frank *et al.*, 2004).

**Table 2**  
*Design Approaches for ULS According to Eurocode 7*

Design Approach	Actions	Soil Parameters	Resistances
DA1-1	A1	M1	R1
DA1-2	A2	M2	R1
DA2	A1	M1	R2
DA3	Struct. <sup>1)</sup> : A1 Geot. <sup>2)</sup> : A2	M2	R3

<sup>1)</sup> Structural action: action from a supported structure applied directly to the wall.

<sup>2)</sup> Geotechnical action: action by the ground on the wall.

**Table 3**  
*Partial Factors for Actions According to Eurocode 7*

Design Approach	Actions, $\gamma_F$	
	Permanent unfavourable <sup>1)</sup> , $\gamma_G$	Variable <sup>2)</sup> , $\gamma_Q$
DA1-1	1.35	1.5
DA1-2	1.00	1.30
DA2	1.35	1.50
DA3	Struct. <sup>3)</sup> 1.35	1.50
	Geot. <sup>4)</sup> 1.00	1.30

<sup>1)</sup> Favourable permanent action:  $\gamma_G = 1.00$ .

<sup>2)</sup> When unfavourable; favourable action should not be considered.

<sup>3)</sup> Structural action: action from a supported structure applied directly to the wall.

<sup>4)</sup> Geotechnical action: action by the ground on the wall.

**Table 4***Partial Factors for Soil Properties and Resistances According to Eurocode 7*

Design Approach	Soil properties, $\gamma_M$			Resistances, $\gamma_R$
	$\tan\phi'$	$c'$	$c_u$	
DA1-1	1.00	1.00	1.00	1.00
DA1-2	1.25	1.25	1.40	1.00
DA2	1.00	1.00	1.00	1.40
DA3	1.25	1.25	1.40	1.00

One problem when using numerical methods arises because DA1-C1 and DA2 require permanent unfavourable actions (*e.g.* the earth pressures acting on retaining structures) to be factored by a partial factor of safety. This is not possible because in numerical analyses the earth pressure is a result of the analysis and not an input (Schweiger, 2005; 2010). However, Eurocode 7 allows for the alternative of factoring effect of the action instead the actions itself, *e.g.* bending moments or strut forces. Thus, numerical methods can be used because the analysis is performed with characteristic loads and characteristic parameters introducing the relevant partial factor at the end of the analysis (Schweiger, 2005; 2010).

Consequently, numerical models can be used for ULS design in the Eurocode framework as it follows (Lees, 2013; Frank *et al.*, 2004; Bauduin *et al.*, 2005; López del Blanco, 2011):

*DA1 Combination 1 and DA2.* All ground parameters and permanent actions enter the analysis with their characteristic values. Variable loads are factored by  $\gamma_Q / \gamma_G = 1.5/1.35 = 1.11$ , in order to account for the difference between the partial factors on variable and permanent actions. The outputs of the effect of the actions (*e.g.* structural forces) are multiplied by the partial factor on unfavourable permanent actions,  $\gamma_G = 1.35$ , in order to obtain their design values.

*DA1 Combination 2 and DA3.* There are essentially two main procedures of introducing material factors in geotechnical ULS design, when using numerical models (Bond & Harris, 2008):

a) *Method 1* (duration factoring). The analysis is performed with design values of actions and of the ground strength parameters right from the start throughout all the construction stages; the obtained values of action effects in structural members (bending moments, shear forces, etc.) are design values.

b) *Method 2* (staged factoring). The analysis is performed with characteristic values of actions and of ground strength parameters throughout all construction stages and design values are only imposed in separate, dedicated ULS checks at critical construction stages.

When analysing the application of the DA in numerical modeling, Schweiger, (2005; 2010), points out that DA1 is basically a combination of DA2 and DA3. This means that the application of a partial safety factor



associated with a particular action, soil strength parameter or resistance is performed either in the same manner as DA2\* or DA3.

### 3.6. Results

The following results were obtained applying the design approaches described above. Only the ULS is considered. It is assumed that the wall installation does not disturb the surrounding soil (the wall is wished-in-place).

In order to establish the embedment depth, a series of analyses were performed with different wall lengths. Starting with an embedment depth of 4.0 m, the length of the wall was increased with steps of 0.5 m until numerical convergence was achieved. The following design values of the embedment depth have been set:  $D = 5.5$  m (for DA1-1 and DA2) and  $D = 6.5$  m (for DA1-2 and DA3).

The analyses assume a simplified construction sequence comprising the following steps:

- a) Initial stress state ( $k_0$  based on the characteristic value for the friction angle,  $\phi' = 30^\circ$ ).
- b) Sheet pile wall wished-in-place, surcharge load and excavation to  $-5.5$  m.

The results are summarized in Table 5 and these ones concludes that methods 1 and 2, applicable to DA3 and to DA1-2, lead to very similar results.

**Table 5**  
*Comparison of Design Bending Moments and Shear Forces for Three Design Approaches*

Design Approach		Design bending moment, $M_d$ , [kN.m]	Design shear force, $T_d$ , [kN]
DA1-C1		$377 \times 1.35 = 509$	$155 \times 1.35 = 209$
DA1-C2	Method 1	601	224
	Method 2	598	219
DA2		$377 \times 1.35 = 509$	$155 \times 1.35 = 209$
DA3	Method 1	633	239
	Method 2	630	237

### 4. Conclusions

Application of the finite element method to ULS design of a simple excavation supported by cantilevered wall investigating all the design approaches according to Eurocode 7 has been presented. The results are reasonably consistent with respect to the design values of the bending moments and shear forces, at least for this particular example.

This paper aims to demonstrate how implement the concept of partial factors of safety, as established in Eurocode 7, in finite element software, to perform geotechnical ultimate limit state design calculations.

## REFERENCES

- Bauduin C., Bakker K.J., Frank R., *Use of Finite Element Methods in Geotechnical Ultimate Limit State Design*. Proc. XVI<sup>th</sup> Int. Conf. on Soil Mech. a. Geotechn. Engng., Osaka, Japan, 2005
- Bond A., Harris A., *Decoding Eurocode 7*, Taylor&Francis, London, 2008.
- Brinkgreve R.B.J., Broere W., Waterman D., *Plaxis, Finite Element Code for Soil and Rock Analyses, Users Manual*. Balkema, Rotterdam, 2006.
- Desai C.S., Christian J., *Numerical Methods in Geotechnical Engineering*, McGraw-Hill, New York, 1977.
- Frank R., Bauduin C., Driscoll R. et al., *Designer's Guide to EN-1997-1 Eurocode 7: Geotechnical Design – General Rules*. Thomas Telford, London, 2004.
- Lees A., *Using Numerical Analysis with Geotechnical Design Codes*. In P. Arnold et al. (Eds.), *Modern Geotechnical Design Codes of Practice*, IOS Press, Amsterdam, 2013, 157-170.
- López del Blanco I., *Numerical Analysis of Embedded Walls for Design with Eurocode 7*. Master Diss., Univ. Politècnica de Catalunya, 2011.
- Popa H., *Interacțiunea între un perete de susținere și fundația unei construcții învecinate. Factori de influență*. Rev. Română de Geotehn. și Fund., 2 (2004).
- Potts D.M., Zdravkovic L., *Finite Element Analysis in Geotechnical Engineering. Theory*, Thomas Telford Ltd., London, 1999.
- Potts D.M., Zdravkovic L., *Finite Element Analysis in Geotechnical Engineering. Application*, Thomas Telford Ltd., London, 2001.
- Schweiger H.F., *Application of FEM to ULS Design (Eurocodes) in Surface and Near Surface Geotechnical Structures*. Proc. XI<sup>th</sup> Int. Conf. Computer Methods and Adv. in Geomech., Bologna, Italy, 2005.
- Schweiger H.F., *Numerical Analysis of Deep Excavations and Tunnels in Accordance with EC7 Design Approaches*. Proc. Int. Conf. Geotechn. Challenges in Megacities, Moskau, 2010.
- \* \* \* *Eurocode 7 Part 1, General Rules CEN/TC 250/SC7. EN 1997-1*, European Committee for Standardization, Brussels, 2004.

APLICAREA METODELOR NUMERICE LA PROIECTAREA DUPĂ EUROCOD A  
STRUCTURILOR DE SPRIJIN ÎNGROPATE

(Rezumat)

Procedeele de calcul curent adoptate în practica proiectării structurilor de sprijin au drept suport analitic formulări bazate pe echilibrul limită al masivelor de pământ. Această abordare este limitativă în informații necesare unei proiectări riguroase. În contradicție cu metodele tradiționale, și ca o alternativă la acestea, soluțiile numerice pot furniza într-o manieră consistentă, datorită posibilităților de modelare pe care le oferă, toate datele necesare unei proiectări riguroase. În lucrare este prezentată modalitatea prin care metodele numerice pot fi aplicate la proiectarea conform Eurocodului 7 a structurilor de sprijin îngropate. De asemenea sunt făcute și unele observații privind modelarea cu elemente finite (alegerea și definirea geometriei modelului, calibrarea etc.).